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The contribution of spin torque to the spin Hall coefficient and spin motive force in a spin–orbit coupling system

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Abstract
We derive rigorously the relativistic angular momentum conservation equation by means of quantum electrodynamics. The novel nonrelativistic spin current and torque in the spin–orbit coupling system, up to the order of $1/c^4$, are exactly investigated using Foldy–Wouthuysen transformations. We find a perfect spin Hall coefficient including the contribution of the spin torque dipole. A novel spin motive force, analogous to the Lorentz force, is also obtained to help us understand the spin Hall effect.

1. Introduction

Spintronics has become a fast developing field since it was first discovered. The aspects of the carriers' spin degree and the spin Hall effect [1–3] concerned with transport have been paid a lot of attention recently. In order to describe the spin transport properly, the definition of spin current has been discussed and various theories of spin current have been established [4, 5]. In a traditional review, the spin current was presented in terms of an anticommutator of the velocity and the spin, $(1/2)\varphi^+\{\mathbf{v}, \mathbf{s}\}\varphi$. However, under such a definition one problem is that there is no conjugate spin force to link the spin current. Therefore, the Onsager relation cannot be established [6]. Furthermore, because the spin has its own dynamics in its Hilbert space, the current with both spin and spatial degree is not conserved due to the spin–orbit coupling. By considering the spin torque, a source in the spin continuity equation can be achieved. Previous investigations of the spin torque depended on the spin relaxation time [7–12]. To our knowledge, an explicit torque beyond an approximation of spin relaxation time has not yet been established.

In the studies of the spin Hall effect, the experiments and theories focus on the spin Hall coefficient $\sigma_{\rm SH}$ [4, 5, 13–31]. In comparison with Ohm's law in electronics, in response to the applied electric field, a spin current j_s^{kl} is generated, $j_s^{kl} = \sigma_{SH}\varepsilon^{lkm}E^m$ [4]. Recent studies show that the spin Hall coefficient σ_{SH} not only includes the contribution of the conventional spin current, but also the torque dipoles which are contained in semiconductor models with the effect of disorder [6, 32]. However, those contributions from the torque dipoles have not been clearly found yet.

Based on the above considerations, the consistency of quantum electrodynamics and Noether's theorem in the derivation of the exact conservation equation for the relativistic angular momentum was suggested [33, 34]. It is found that the spin current including a correction is different from the traditional definition. In the application the spin Hall conductivity σ_{SH} involving the correction can be obtained. Under the requirement of the Onsager relation the spin force is found to relate to the spin Hall coefficient, therefore, relating the topological aspect of systems with spin–orbit coupling.

2. Spin continuity equation

Let us firstly consider the relativistic Lagrangian with Dirac fields Ψ and $\bar{\Psi}$ coupled to an electromagnetic field A^μ, $\mathcal{L} = \mathcal{L}_D + \mathcal{L}_{em} + \mathcal{L}_{int}$, where $\mathcal{L}_D = \bar{\Psi}(i\hbar c\gamma^\mu\partial_\mu - mc^2)\Psi$ describes the free Dirac fields of spin 1/2, $\mathcal{L}_{em} = -(1/4)F^{\mu\nu}F_{\mu\nu}$ is the Lagrangian of the electromagnetic field, where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, the interaction between Dirac fields and the electromagnetic field is given by $\mathcal{L}_{int} = -e\bar{\Psi}\gamma^\mu A_\mu\Psi$, and the four-vector γ^μ is represented as $\gamma^\mu = (\gamma^0, \gamma)$ in terms of Pauli matrices σ.

The energy–momentum tensor of gauge invariant form is found to be $\theta^{\mu\nu} = \theta_D^{\mu\nu} + \theta_{em}^{\mu\nu} + \theta_{int}^{\mu\nu}$, where $\theta_D^{\mu\nu} = \bar{\Psi}i\hbar c\gamma^\mu\partial^\nu\Psi - g^{\mu\nu}\mathcal{L}_D$, $\theta_{em}^{\mu\nu} = -F^{\mu\sigma}\partial^\nu A_\sigma - g^{\mu\nu}\mathcal{L}_{em}$, and $\theta_{int}^{\mu\nu} = -g^{\mu\nu}\mathcal{L}_{int}$. Here $g^{\mu\nu} = g_{\mu\nu}$ is the metric tensor with $g^{00} = 1$, $g^{ii} = -1$ ($i = 1, 2, 3$), and $g^{\mu\nu} = 0$ ($\mu, \nu = 0, 1, 2, 3, \mu \neq \nu$). This energy–momentum tensor satisfies the conservation law, i.e. $\partial_\mu\theta^{\mu\nu} = 0$. With the tensor the angular momentum tensor can be written in the form of $M^{\alpha\mu\nu} = s^{\alpha\mu\nu} + l^{\alpha\mu\nu}$. Here $l^{\alpha\mu\nu} = x^\mu\theta^{\alpha\nu} - x^\nu\theta^{\alpha\mu}$ is the orbital angular momentum tensor and $s^{\alpha\mu\nu} = s_D^{\alpha\mu\nu} + s_{em}^{\alpha\mu\nu}$ is spin angular momentum tensor, where $s_D^{\alpha\mu\nu} = (\partial\mathcal{L}/\partial\partial_\alpha\Psi)I_D^{\mu\nu}\Psi$ and $s_{em}^{\alpha\mu\nu} = (\partial\mathcal{L}/\partial\partial_\alpha A_\sigma)(I_{em}^{\mu\nu})_{\sigma\rho}A^\rho$. Considering the notations $I_D^{\mu\nu} = -\mathrm{i}\sigma^{\mu\nu}/2$ and $(I_{em}^{\mu\nu})_{\sigma\rho} = g_\sigma^\mu g_\rho^\nu - g_\rho^\mu g_\sigma^\nu$, it is found that $s_D^{\alpha\mu\nu} = \mathrm{i}(\hbar c/4)\bar{\Psi}\gamma^\alpha[\gamma^\mu, \gamma^\nu]\Psi$ and $s_{em}^{\alpha\mu\nu} = A^\mu F^{\alpha\nu} - A^\nu F^{\alpha\mu}$. The corresponding conservation law for the total angular momentum is $\partial_\alpha M^{\alpha\mu\nu} = 0$.

In order to obtain the nonrelativistic form of the conservation law, the Foldy–Wouthuysen transformation is used in the following calculations up to $1/c^4$. The nonrelativistic wavefunction is written in terms of a transformation on the relativistic wavefunction Ψ, $\Psi'' = \exp[\mathrm{i}s'(\alpha)]\exp[\mathrm{i}s(\alpha)]\Psi$, where the operators in the exponential are $\mathrm{i}s(\alpha) \equiv (\beta/2mc)\alpha\cdot\pi$ and $\mathrm{i}s'(\alpha) \equiv (\mathrm{i}\hbar e/4m^2c^3)\alpha\cdot\mathbf{E}$. Here $\mathbf{E}$ is the electric field intensity. Correspondingly the wavefunction is written in the form as $\Psi'' = (\varphi'', \chi'')^{\mathrm{T}}$, where $\varphi'' = [1 - s(\sigma)^2/2\beta^2]\varphi$ and $\chi'' = [\mathrm{i}s'(\sigma) - \mathrm{i}(E - e\phi)s(\sigma)/2mc^2\beta - \mathrm{i}s(\sigma)^3/3\beta^3]\varphi$. Introducing the notation $\eta = \mathrm{i}\hbar e\sigma\cdot\mathbf{E} - (E - e\phi)\sigma\cdot\pi - (\sigma\cdot\pi)^3/6m$, χ'' is presented as $\chi'' = (\eta/4m^2c^3)\varphi$. With the help of the formula $\mathrm{e}^{\mathrm{i}s(\alpha)}\hat{O}\mathrm{e}^{-\mathrm{i}s(\alpha)} = \hat{O} + [\mathrm{i}s, \hat{O}] + [\mathrm{i}s, [\mathrm{i}s, \hat{O}]]/2 + \cdots + [\mathrm{i}s, [\mathrm{i}s, \ldots, [\mathrm{i}s, \hat{O}]\cdots]]/n! + \cdots$ and let $\hat{O}$ be M^{0ij} and M^{kij}, the continuity equation for the nonrelativistic electronic spin can be obtained. The nonrelatistivic form of the angular momentum conservation law reads

$$\frac{\partial}{\partial t}\rho_s^l + \nabla^k j_s^{kl} = T^l, \tag{1}$$

where $j_s^{kl} = (\hbar/4m)\varphi^\dagger\{\pi^k, \sigma^l\}\varphi$ is the traditional spin current, which represents the current of the l component of the spin along the direction k. Here we have written the wavefunction φ'' as φ for convenience. The spin density ρ_s^l is obtained as

$$\rho_s^l = \frac{\hbar}{2}\varphi^+\sigma^l\varphi + \frac{\hbar}{4m^2c^2}\varphi^+(\pi^l\sigma\cdot\pi - \pi^2\sigma^l)\varphi$$
$$+\frac{\hbar^2 e}{8m^2c^3}\varphi^+(3B^l - \sigma^l\sigma\cdot\mathbf{B})\varphi + \frac{\mathrm{i}\hbar}{8m^3c^4}\varphi^+[(\sigma\times\pi)^l\eta - \eta^+(\sigma\times\pi)^l]\varphi, \qquad (2)$$

where magnetic field **B** is evidently written out. The first term in equation (2) is nothing but a traditional spin density. The second term can be written as $(\hbar/4m^2c^2)\varphi^+\pi\times(\pi\times\sigma)\varphi$, which indicates its generation from the spin–orbit coupling. The interaction between the intrinsic magnetic moment and the external magnetic field is given by the third term. The last term gives a small correction of the order of $1/c^4$.

Now let us analysis the right-hand side of equation (1), named the spin torque density T^l. Up to the same order of the nonrelativistic approximation, it is found that

$$T^l = \nabla^k\left\{\frac{\mathrm{i}\hbar}{2m}\varphi^+\sigma^k(\sigma\times\pi)^l\varphi\right\} + \frac{\mathrm{i}\hbar e}{2mc}\varphi^+[\sigma^l\sigma\cdot\mathbf{B} - B^l]\varphi$$
$$-\frac{\hbar e}{4m^2c^2}\varphi^+\{\hbar[\nabla(\mathbf{E}\cdot\sigma)\times\sigma]^l + 2\sigma^l\pi\cdot\mathbf{E} - 2\sigma\cdot\pi\ E^l\}\varphi$$
$$+\frac{\hbar^2 e}{4m^2c^2}\nabla^k\{\varphi^+[\sigma^k(\sigma\times\mathbf{E})^l + (\sigma\times\mathbf{E})^k\sigma^l]\varphi\}$$
$$-\frac{1}{32m^4c^4}\varphi^+(\eta^+\{\sigma\cdot\pi,\{\sigma\cdot\pi,(\pi\times\sigma)^l\}\} + \{\sigma\cdot\pi,\{\sigma\cdot\pi,(\pi\times\sigma)^l\}\}\eta)\varphi$$
$$+\frac{\hbar}{64m^4c^4}\nabla^k[\varphi^+(\eta^+\{\sigma\cdot\pi,\{\sigma\cdot\pi,\sigma^k\sigma^l\}\} + \{\sigma\cdot\pi,\{\sigma\cdot\pi,\sigma^k\sigma^l\}\}\eta)\varphi]. \qquad (3)$$

Besides the relativistic correction up to the order of $1/c^4$, the contributions from the spin–orbit coupling and its nonrelativistic correction are presented by the first and the fourth terms. The second term corresponds to the interaction of intrinsic magnetic moment and external magnetic field. The effect from the couplings between the orbit and the spin to the electric field is given in the third term.

Previous discussion of the spin Hall effect was given in the case of the absence of the magnetic field **B**. In general, to extend the cases for the ferromagnet or the system under the external magnetic field, the magnetic field remains in the following and demonstrates the effect of magnetic field on the spin Hall effect. Considering an external magnetic field along the direction of the spin, one state of the spin polarization is left and all spin transport processes in the presence of both the electric field E^m and the magnetic field B^l are shown in figure 1(b). The corresponding spin motive force and the spin Hall coefficient can be obtained. It is worth pointing out that the previous spin current does not contain the contribution of spin torque dipole [6]. When the torque density is written in the form of a divergence of a torque dipole $T^l = -\nabla^k P_T^{kl}$, where $P_T^{kl} = \int_v T^l\,\mathrm{d}x^k$ is integrable, the spin current is found

$$J_s^{kl} = j_s^{kl} + P_T^{kl}, \qquad (4)$$

which includes the traditional current and a correction of the spin torque dipole. Equation (4) can be written as a response equation $J_s^{kl} = \sigma_{sc}\varepsilon^{lkm}E^m$ in which σ_{sc} is the spin Hall coefficient. Obviously, the spin current J_s^{kl} is vertical to the direction of the spin s^l and the electric field E^m. E^m, s^l, and J_s^{kl} satisfy the right-hand rule, as shown in figure 1(a).

Now the spin continuity equation (1) can be written as

$$\frac{\partial}{\partial t}\rho_s^l + \nabla^k J_s^{kl} = 0. \qquad (5)$$

It implies that the spin current has a natural conjugate spin force. Therefore, the Onsager relation $\sigma_{sc}^{mk} = -\sigma_{cs}^{km}$ can be established under the time reversal symmetry to link the spin

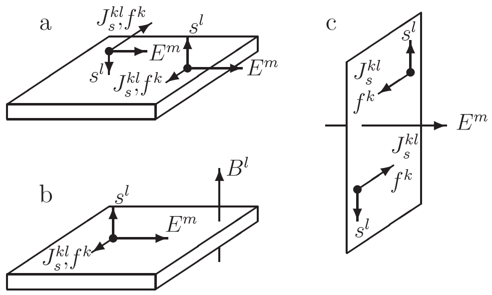

Figure 1. The spin current J_s^{kl} and the spin motive force f^k via the spin **s** and the electric field **E**, where J_s^{kl} represents the current of the l component s^l of the spin along the direction k. (a) The spin current J_s^{kl} and the spin motive force f^k in the spin–orbit coupling system without an external magnetic field, where E^m, s^l and J_s^{kl} (or f^k) satisfy the right-hand rule; (b) the spin current and the spin motive force in the spin–orbit coupling system under an external magnetic field **B** along the l direction; (c) the spin current and the spin motive force in the two-dimensional Rashba spin–orbit coupling system.

transport with other transport phenomena, such as the charge transport, where $\sigma_{\mathrm{sc}}^{mk}$ and $\sigma_{\mathrm{cs}}^{km}$ are the spin–charge and charge–spin conductivity tensors.

3. Spin Hall coefficient and spin motive force

We consider the divergence of the spin torque dipole as a product of an electric field and a coefficient $\chi^{lm}(\mathbf{q})$, $-\mathrm{i}q^k P_{\mathrm{T}}^{kl}(\mathbf{q}) = \chi^{lm}(\mathbf{q})E^m(\mathbf{q})$, with $\mathbf{q}$ being a finite wavevector. The more explicit form of the coefficient can be represented as follows

$$\chi^{lm} = -\frac{\hbar}{2}\varepsilon^{lm'm}q^{m'}\frac{\sigma_{\mathrm{e}}}{e} + \frac{\mathrm{i}\hbar e}{2mc}\varphi^{+}(\mathbf{q})[(\sigma^l\sigma\cdot\mathbf{B} - B^l)/E^m]\varphi(\mathbf{q}) - \frac{\hbar e}{4m^2c^2}\varphi^{+}(\mathbf{q})(\mathrm{i}\hbar q^{m'}\sigma^m\sigma^{n'}\varepsilon^{lm'n'} + 2\sigma^l\pi^m)\varphi(\mathbf{q}), \tag{6}$$

where σ_{e} is the electric conductivity. The spin Hall coefficient σ_{sc} corresponding to our new spin current J_s^{kl} can be written as

$$\sigma_{\mathrm{sc}} = \sigma_{\mathrm{SH}}^0 + \sigma_{\mathrm{SH}}^{\mathrm{T}}, \tag{7}$$

where σ_{SH}^0 is the conventional spin Hall conductivity [4, 12], corresponding to the traditional spin current, $\sigma_{\mathrm{SH}}^{\mathrm{T}}$ is the contribution of the spin torque dipole P_{T}^{kl}, and $\sigma_{\mathrm{SH}}^{\mathrm{T}} = \mathrm{Re}\{\mathrm{i}\partial\chi^{lm}(\mathbf{q})/\partial q^k\}_{\mathbf{q}=0}$. In some semiconductors with disorder the spin Hall coefficient is extremely different from the conventional one. We can evaluate the spin Hall coefficient $\sigma_{\mathrm{SH}}^{\mathrm{T}}$ in the GaAs sample as follows: at room temperature, the carrier density of GaAs is $n \sim 10^{17}$ cm^{-3}, the mobility of carriers is $\mu \sim 350$ cm^2 V^{-1} s^{-1}, and the conventional spin Hall coefficient is $\sigma_{\mathrm{SH}}^0 \sim 16\ \Omega^{-1}$ cm^{-1}, $\sigma_{\mathrm{SH}}^{\mathrm{T}} \sim 5.6\ \Omega^{-1}$ cm^{-1}. For a lower carrier density case, $n \sim 10^{16}$ cm^{-3}, $\mu \sim 400$ cm^2 V^{-1} s^{-1}, $\sigma_{\mathrm{SH}}^0 \sim 7.3\ \Omega^{-1}$ cm^{-1}, $\sigma_{\mathrm{SH}}^{\mathrm{T}}$ is estimated as $\sigma_{\mathrm{SH}}^{\mathrm{T}} \sim 0.64\ \Omega^{-1}$ cm^{-1}. As a kind of correction, $\sigma_{\mathrm{SH}}^{\mathrm{T}}$ is one order smaller than the conventional spin Hall coefficient σ_{SH}^0. The general spin Hall coefficient σ_{sc} should include the conventional one σ_{SH}^0 and the correction $\sigma_{\mathrm{SH}}^{\mathrm{T}}$.

Now the Onsager relation and spin Hall coefficient have been found. The so-called spin force $\mathbf{F}_{\mathrm{s}}$ can be calculated as $\mathbf{F}_{\mathrm{s}} = (\mathbf{J}_{\mathrm{c}} - \sigma_{\mathrm{cc}}\mathbf{E})/\sigma_{\mathrm{cs}}$, where σ_{cc} is the charge–charge conductivity tensor, and $\mathbf{J}_{\mathrm{c}}$ is the charge current [6]. Particularly, in [12], the spin force has the simple form $F_{\mathrm{s}}^m = J_{\mathrm{c}}^k/\sigma_{\mathrm{cs}}^{km}$ in a two-dimensional electron gas. From the Onsager relation, $\sigma_{\mathrm{cs}}^{km} = -\sigma_{\mathrm{sc}}^{mk}$, the charge–spin tensor σ_{cs}^{km} can be obtained, and the intrinsic Hall current J_c^k in the k direction can be detected by experiments. However, the spin force cannot be interpreted as a motive force of an electron like the Lorentz force in the Hall effect, and it has the same direction as the electric field E^m.

To interpret the spin Hall effect, we try to find a spin motive force f^k which has an analogy to the Lorentz force in the Hall effect. Here the spin motive force is vertical to the direction of the electric field and the spin, i.e. E^m, s^l and f^k satisfy the right-hand rule, as shown in figure 1(a). The discussion is based on the spin torque. The torque density T^l can be written in the form $T^l = \varepsilon^{lmk} r^m f^k = \chi^{lm} E^m$. After calculation, we obtain f^k as

$$f^k = \sigma_f^1 E^m + \sigma_f^2 \chi^{lm}, \tag{8}$$

where the spin motive force coefficients σ_f^1 and σ_f^2 are expressed as

$$\sigma_f^1 = \tfrac{1}{2}\,\mathrm{Re}\{\varepsilon^{lmk} \nabla^m \chi^{lm}(\mathbf{r})\} \tag{9}$$

and

$$\sigma_f^2 = \tfrac{1}{2}\,\mathrm{Re}\{\varepsilon^{lmk} \nabla^m E^m(\mathbf{r})\}. \tag{10}$$

In the case of the electric field being constant, σ_f^2 is zero. Here we have obtained the evident formula χ^{lm}, and the electric field E^m can be detected in experiments. Thus the spin motive force f^k is found. Assuming the mobility of the carriers in the GaAs sample with disorder is $\mu \sim 10^3\ \mathrm{cm^2\ V^{-1} s^{-1}}$ and the electric field is $\mathbf{E} \sim 10\ \mathrm{mV}\ \mu\mathrm{m}^{-1}$, we find the order-of-magnitude of the spin motive force $f^k \sim 10^{-20}\ \mathrm{eV}\ \mu\mathrm{m}^{-1}$. Obviously, this is an extremely weak quantity.

4. Application in the two-dimensional electron gas

We will discuss the properties of the spin motive force in the two-dimensional electron gas. The Dirac Hamiltonian of a relativistic electron is $H = c\alpha \cdot \mathbf{P} + \beta mc^2$. Using the Foldy–Wouthuysen transformation, the nonrelativistic limit of the Dirac Hamiltonian is $H = \beta(mc^2 + \pi^2/2m - \pi^4/8m^3c^2) + e\phi - (\hbar e/2mc)\beta\sigma \cdot \mathbf{B} - (\hbar^2 e/8m^2c^2)\nabla \cdot \mathbf{E} - \mathrm{i}(\hbar^2 e/8m^2c^2)\sigma \cdot (\nabla \times \mathbf{E}) - (\hbar e/4m^2c^2)\sigma \cdot (\mathbf{E} \times \mathbf{P})$, where ϕ is the electric potential [34]. In the two-dimensional electron gas, $\mathbf{E} = (0, 0, E^m)$, $\sigma = (\sigma^l, \sigma^k, \sigma^m)$, $\mathbf{P} = (P^l, P^k, 0)$, and $\mathbf{B} = 0$, the nonrelativistic Hamiltonian can be written as $H = \mathbf{P}^2/2m - \lambda(P^k\sigma^l - P^l\sigma^k)$, this is the Rashba Hamiltonian, where the coupling parameter $\lambda = (\hbar e/4m^2c^2)E^m$ [35].

In the two-dimensional electron gas, the formula χ^{lm} has a simple form, $\chi^{lm} = \mathrm{i}(\hbar/2e)\varepsilon^{lm'm}\nabla^{m'}\sigma_e - (\hbar e/4m^2c^2)\varphi^+(\hbar\nabla^{m'}\sigma^m\sigma^{n'}\varepsilon^{lm'n'} + 2\sigma^l\pi^m)\varphi$. Thus the spin motive force can be represented as $f^k = (\varepsilon^{lkm}\hbar e/8m^2c^2)\nabla^m[\varphi^+(\hbar\nabla^{m'}\sigma^m\sigma^{n'}\varepsilon^{lm'n'} + 2\sigma^l\pi^m)\varphi]E^m$. The spin motive force f^k is nonzero, and it induces the spin current, so the spin Hall effect can be observed in experiments in the two-dimensional electron gas. In this case, f^k should be vertical to the spin s^l and electric field E^m, as shown in figure 1(c). In [36], the author introduced a spin transverse force which is perpendicular to the spin current. In contrast, our spin motive force is parallel to the spin current. So it can be used to better understand the mechanism of the spin Hall effect.

5. Conclusion

In conclusion, we induce the spin continuity equation from the angular momentum conservation law with spin–orbit coupling. Our results naturally include a correction to the traditional spin current. The correction could be considered as a spin torque dipole, so there is a conjugate force linking the spin current, and the Onsager relation can be established. A perfect spin Hall coefficient corresponding to the new spin current is conformed. Furthermore, the magnitude of the spin Hall coefficient is evaluated. From the explicit spin torque, we introduce a spin motive force, having the same direction as the spin current, to better understand the spin Hall effect. We find a novel right-hand rule between the electric field, the spin, and spin current (or spin motive force) in spintronics.

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